ISI – Bangalore Center – B Math - Physics IV – End Semestral Exam
Date: 20 April 2016. Duration of Exam: 3 hours
Total marks: 50

Q1. [Total Marks: 2x5=10]

Please give SHORT answers to the questions below.

a.) Two clocks at two ends of a train compartment are synchronized with respect to the train frame. The train is moving with a constant velocity with respect to the ground in the positive x direction. To an observer at rest on the ground, will the clock at the front of the compartment show a later or an earlier time compared to the clock at the back?

b.) Give an example to show that addition of two time like vector need not be a time like vector. What kind of time like vectors can be added to always produce a time like vector?

b.) An particle of mass m is subject to a potential $V(x) = \infty$ if $|x| > a/2$, and $V(x) = 0$ otherwise. Assume that $\psi(x,t=0) = A$ for $0 < x < a/2$, $\psi(x,t=0) = -A$ for $-a/2 < x < 0$.

If after some time the energy of the particle is measured, which of the following energies will NOT be found? i.) $\frac{18\hbar^2\pi^2}{ma^2}$, ii.) $\frac{25\hbar^2\pi^2}{2ma^2}$ [Hint: The lowest energy state has energy $\frac{\hbar^2\pi^2}{2ma^2}$]

c.) The parity operator is defined as $P\psi(x,y,z) = \psi(-x,-y,-z)$. Is Parity operator a Hermitian Operator? Give a short proof or a counter example.

d.) For a three dimensional quantum mechanical system with central potential V(r), the operator L_z commutes with the Hamiltonian. Does this imply that energy eigenstates are also eigenstates of L_z? Justify your answer.

e.) Assume that for a 3 dimensional quantum system each of the three components of the angular momentum operator commute with the Hamiltonian. How many of these components can be simultaneously and accurately measured? Justify your answer.

Q 2. [Total Marks: 5+5=10]

a.) A particle has the velocity u_x, u_y with respect to the ground frame. A person walks with velocity v with respect to the ground along the x direction. What must v be (other than zero) so that the person finds that the particle has the same transverse speed u_y?

b.) Show that it is not possible for a free electron to spontaneously absorb or emit a photon. [Hint: show that it will violate energy-momentum conservation. You can take c=1 for convenience of calculation.]

Q3. [Total Marks: 10]

a.) Show that solution to the general Schrodinger equation for a free particle can be expressed as

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi_k e^{i(kx - \frac{\hbar k^2}{2m}t)} dk .$$

Assume that a free particle has an initial wave function given by $\psi(x, t=0) = \frac{1}{\sqrt{a}} e^{ik_0 x}$ for $-\frac{a}{2} \leq x \leq \frac{a}{2}$ and zero otherwise. Answer the following for a free particle with this initial wave fnction.

b.) Determine ϕ_k .

c.) Determine the probability distribution function *P(k)* for measuring momentum between $\hbar k$ and $\hbar(k + dk)$. Plot *P(k)*.

e.) What is the value of momentum that has the highest probability of being measured?

f.) Show that the probability of finding the particle with energy $\frac{\hbar^2}{2m}(k_0 + \frac{2n\pi}{a})^2$ is zero.

Q4. [Total Marks: 3+1+2+2+3=10]

A Hydrogen atom is known to be in a such a state so that it has equal probability to be found in the ground state $n=1$, $l=0$ or in the first excited state $n=2$, $l=1, m=0$. Remember that the Hydrogen atom is a quantum mechanical system with a central potential, in which each component of the angular momentum operator commutes with the Hamiltonian operator.

a.) Show that the measured spread in energy ΔE in this state will be equal to (3/8)th of ground state energy.

b.) What is the value of $< L^2 >$ in this state?

c.) Show that $< L_x > = < L_y > = < L_z > = 0$ in this state.

d.) Why are the expectation values in b.) and c.) independent of time?
e.) Prove that for any normalizable state of Hydrogen atom $< L^2 > \geq (< L_z >)^2$.

Q5. [Total Marks: 3+3+4=10]

The energy eigenstates of a one dimensional harmonic oscillator is given by
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2 \psi = E\psi .$$

a. Prove that there is no normalizable solution if E<0.

b. Analyze the equation in the region where |x| is large, and show that the asymptotic form of a bound state wave function must be proportional to $e^{-\xi^2/2}$ where $\xi = \sqrt{\frac{m\omega}{h}}x$.

c. Prove that energy eigenstates of a one dimensional harmonic oscillator are non degenerate.